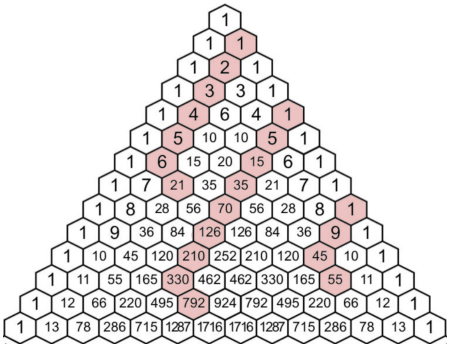
Math 218: Elementary Number Theory Spring 2025

The Basics



Instructor: Dr. Christy Hazel (she/her)[1]
Email: hazelchristy@grinnell.edu
Office: Noyce 2248
Drop-in Office Hours: M 2:20-3:50PM, F 10:30-11:30AM
Appointments: See link PWeb to schedule an appointment

Class Mentor: Ian Clawson
Mentor Email: clawsoni@grinnell.edu

Class Meetings: TTh 8:30–9:50AM in Noyce 2245

Required Textbooks: *Explorations in Number Theory* by Jeanne Agnew (free on PWeb under "Course Reserves")
Discrete Structures by Kyle Ormsby and David Perkinson (free on Dr. Ormsby's website)
Optional Textbook: *A Friendly Introduction to Number Theory* by Joseph H. Silverman

What is Number Theory?

Number theory, in its most basic form, studies patterns within the positive integers:

$$1, 2, 3, 4, 5, 6, 7, 8, 9, \ldots$$

The subject is especially interested in the relationships between these numbers. For example, how are the prime numbers $(3, 5, 7, 11, 13, 17, \ldots)$ distributed amongst the positive integers? Or given a particular polynomial equation, does it have integer solutions? And if it does, how do we best describe those integer solutions? As you might observe, many of the questions asked in elementary number theory are simple to state. However, as we develop the tools needed to answer these questions, we will discover that the answers and methods can be quite deep and mysterious.

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[1]Feel free to call me Christy, or if you prefer something more formal, Professor Christy, Professor Hazel, Dr. Christy, Dr. Hazel are all fine!

Course Goals

The primary goal of this course is to develop your proof-writing skills and to expand your creative problem-solving strategies. We will use number theory as a backdrop, but the main objective is to prepare you for the 300-level foundations courses. Even if you decide not to pursue mathematics after this course, many of these skills are helpful in other fields.

A successful student in this course will...

- Strengthen technical writing and communication skills, especially in mathematics.
- Hone problem-solving skills. In particular, learn how to approach problems that you have never seen before, including those in a new mathematical area.
- Learn about mathematical induction, equivalence relations, and basic counting techniques. (These are foundational topics that will appear in many mathematics courses.)
- Learn about the key concepts in elementary number theory including divisibility, congruences, modular arithmetic, the fundamental theorem of arithmetic, Fermat's little theorem, and quadratic residues.

Note the style of mathematical writing is likely different from what you are used to. Most people feel uncomfortable and unsure when they first start writing proofs–*this is completely okay and expected!* One of our course goals is to learn to work through this discomfort and to become more confident in our proof-writing skills.

Grading Scheme: How Your Learning Will be Assessed

Course grades will be weighted according to the following scheme:

- **Weekly Homework** 23%
- **Reading Reflections** 4%
- **In-Class Engagement** 4%
- **In-Class Worksheets** 4%
- **Two Best Midterm Exams** 30% (15% each)
- **Lowest Midterm Exam** 10%
- **Final Exam** 25%

More information about these categories is given later on in the syllabus.

How to Contact Me

- **Drop-in Office Hours**. These sessions are dedicated time for my students to ask me questions about homework and course material. These will be held in my office (Noyce 2248).
  - Come by any time during the scheduled time.

  - Students should come in one at a time or in groups to ask questions (if you do come in as group, just make sure everyone in the group is comfortable with that).
  - If there is a line, these questions will be limited to 5–10ish minutes to allow the next students to come in to ask questions. You are welcome to get back in line if you have more questions.

- **Appointments**. If you can't make office hours or have a question you'd like to ask outside of drop-in hours, you can schedule a 15-minute appointment with me using the Calendly link on PWeb. Each student is allowed to schedule at most one appointment per week. Please do not select multiple appointments in a given week.
- **Email**. You are welcome to email me any time with any questions or comments about mathematics or about the course (hazelchristy@grinnell.edu).
  - Note I usually *do not* check my email between 6PM and 8AM on weekdays, and I rarely respond to emails on the weekends. You are welcome to email me whenever is most convenient for you, but I usually won't respond until normal working hours.
  - I will try to always respond within one business day (note if you email Friday afternoon, there's a chance I won't get back to you until Monday).

More About Assessments

Homework

Homework will be assigned each week on PWeb. Each assignment will have two parts. The first part of the assignment will be due on **Wednesday at 11:59PM** and be submitted through Gradescope. The second part will be due in class on the following **Tuesday at 8:30AM** if you handwrite your assignment. You must turn in a hard copy at the start of class. If you choose to type your assignment using LaTeX than you can email me a pdf of the assignment by **Tuesday at 4PM**.

The first part of the assignment will consist of one to three proof problems and will only be graded for completion. I will provide feedback on these proofs by Friday at 10AM, and then you will be expected to revise and resubmit these problems as part of your Tuesday assignment. *The second part* will contain rewrites of your first proofs and also include additional proof problems. The second part will be graded on accuracy.

The homework is structured in this way to improve your revision skills. A very common concern students have at this level is: "*I wrote something that sounds like a proof, but I'm not sure if it's complete. How can I tell if what if I've written is correct?*" I want you to learn how to critically analyze your own proof-writing to help alleviate this concern. Regular revisions will help with this goal.

Please see the Late Homework Policy and Academic Honesty sections for more on homework policies.

Reading Reflections

Each week that we don't have an exam you'll have an assigned reading with some reflection question(s). The reading will be related to content in this course, mathematical writing, or mathematics

in general. These will be mostly graded on completion. You can find the assignments on PWeb. **Your lowest reading reflection grade will be dropped.** Late reflections will not be accepted. Instead the drop policy allows everyone to miss one reading reflection with no penalty, no questions asked. With that said, if something is going on that is causing you to miss multiple deadlines, then please reach out to me so we can discuss your options in the course.

In-Class Engagement

An essential component of learning mathematics is regularly and thoughtfully engaging with the material. Our class meetings give us scheduled time to engage with content as a group. This part of your grade will be based on your engagement in class. If you do all of the things listed below, then you will get full points in this category:

- show up on time to class;
- pay attention during the lecture portion of class, taking notes as needed;
- engage with your assigned partner(s) during discussion times and display good group work skills: sharing ideas and questions, listening to ideas, not interrupting your partner(s), etc.;
- stay in the classroom during class time (it's okay to step out every once-in-a-while to run to the bathroom, but you shouldn't be leaving on a regular basis unless you have accommodations);
- refrain from taking out your phone, laptop, outside work, or anything else that would distract you or others during our class time.

You can harm your engagement score by skipping class, showing up late, or being on your cell phone or laptop during class. **Cell phones and laptops should not be used during class**. If you like to use a tablet to take notes, then that's fine, but keep the tablet in note-taking mode for the duration of class.

I do understand life happens, and you might need to occasionally miss class. If you miss two or less classes during the semester, no explanations needed, then your score won't be affected. If possible, let me know beforehand if you are missing class so I can avoid partnering you with anyone.

If you miss more than two classes, then you should expect your engagement score to decline. Documented illnesses/emergencies and university-approved travel are exceptions to this rule. If you are missing more than a week of classes for whatever reason, then please reach out to me so we can come up with a plan to make sure you don't fall behind.

Midterm Exams

We will have three in-class midterm exams:

- Exam 1: **Thursday, February 20**
- Exam 2: **Thursday, March 27**
- Exam 3: **Thursday, April 24**

I will post information about each exam on PWeb the week before the exam. Your lowest exam score will count for 10% of your final grade, while your highest two exam scores will count for 15% of your final grade.

Make-up exams will only be given in the case of documented illnesses or university-approved travel. See Make-up Exam Policy for more information.

Final Exam

We will have a cumulative final exam on **Wednesday, May 14th** from 9am–12pm. More information about the exam will be given on PWeb the week before the exam.

Late Homework Policy

Each student will be allowed to take one 48-hour homework extension for the second part of one homework assignment. Please send me a short email letting me know you are taking your extension, and I will give you instructions for how to submit the late assignment. You do not need to provide a reason why. There is no penalty for taking your one extension.

After this first extension, each student will be allowed to take one more 48-hour extension on the second part of an assignment, but this will result in a 10% grade deduction on that assignment. Again, please send me a short email letting me know you would like a second extension. You do not need to provide a reason why.

No extensions will be given after these two extensions except in extreme, documented situations. No extensions will be given for the first part of the assignment.

If you are struggling to meet deadlines in this course, then I encourage you to make an appointment with me so we can come up with a plan to help you through the semester.

Make-up Exam Policy

If you have university-approved travel (such as for athletics) that conflicts with an exam, then please let me know and provide documentation as soon as possible. Documentation must be provided **at least one week** in advance. In that case, we will schedule a make-up exam for before you leave.

If you are sick or experience a medical emergency on an exam date, then a make-up exam will be provided if you email me a signed doctor's note that states you are/were unable to attend classes on that date. It is unlikely that I will offer a make-up without any documentation. If you have some other emergency that is preventing you from attending an exam, then please reach out as soon as possible to see if a make-up is possible. In any case, please **do not expect a make-up exam if you do not provide documentation stating you had to miss the exam**.

Classroom Culture and Expectations

We will be spending a lot of time discussing mathematics together this term. We should all recognize that different people will have different mathematical and personal backgrounds. To foster and maintain a productive and inclusive learning environment, it is essential that we create a space where people feel comfortable asking questions and discussing material. Here are a few concrete things we can do to build our learning community:

- **Be present and engaged.** We have a limited amount of time together in the classroom, so let's use this time productively. For all of us, this means showing up on time, being ready to start at the beginning of class, and not engaging in distracting behavior such as using phones, looking at non-course-related materials, chatting about unrelated topics during class, leaving during class, etc.
- **Recognize that learning is a process and be open to making mistakes.** I do not expect you to be able to do every proof on your first try. Be open to trying new problems and making mistakes. Recognize that mistakes are a natural part of learning, and find something to learn from your mistakes.
- **Listen to understand, not to judge.** You will often be discussing mathematics with your classmates. Make sure everyone gets a chance to share ideas, and listen to understand and learn from your classmates' ideas, not to judge their abilities. If you disagree with something, share your disagreement respectfully and make sure you are critiquing the ideas and not the person.
- **Take space and give space.** "Take space" by participating in discussions. Ask questions and share ideas. We all benefit from hearing different ideas and thought processes. Also make sure you "give space". If you find that you are dominating a conversation or are the only person in class answering questions, then ask others what they think or wait for someone else to raise their hand. Take time to listen (and actually listen–don't just wait to share your own ideas). **Creating an inclusive and productive classroom requires all of us to be mindful of the space we occupy.**

Class Mentor

Our course mentor, Ian Clawson, will also be available to talk about proofs from class or the readings. He can help with homework and your writing, particularly by pointing out where a completed solution goes wrong, or where a written solution gets hard to follow. Ian will have regular mentor sessions. The time and location which will be posted on PWeb.

Grinnell Resources for Support and Learning

Below are some of the resources offered by Grinnell that could be helpful for you. Grinnell has a lot of resources available for students, so it can be hard to stay informed about all of them. The link https://www.grinnell.edu/students has more information on resources.

Academic Accommodations Based on a Disability

Everyone in this class deserves an equitable opportunity to learn and engage with the material. Students with documented disabilities that are working with the Office of Disability Resources should have accommodation letters available to faculty via the accommodation portal. To help ensure that your access needs are met, I encourage individual students to approach me so we can have a discussion about your distinctive learning needs and accommodations within the context of this course. If you have not already worked with the Office of Disability Resources and believe you may require academic accommodations for this course, Disability Resources staff can be reached via email at access@grinnell.edu, by phone 641-269-3089, or by stopping by their offices on the first floor of Steiner Hall. If you have any other questions about accommodations in this course, please feel free to reach out to me.

Counseling and Wellness

Student Health and Wellness (SHAW) provides a variety of services to support the physical, mental, and social well-being of Grinnell students. Find more information by visiting their website (https://www.grinnell.edu/about/leadership/offices-services/student-health/counseling), including information about counseling services that are available to Grinnell students. They also offer a Need-to-Talk 24/7 counseling hotline at 641-269-4404. These calls will offer a one-time session to work through a specific concern.

Religious Observance

I encourage students who plan to observe holy days that coincide with class meetings or assignment due dates to consult with me in the first three weeks of classes so that we may reach a mutual understanding of how you can meet the terms of your religious observance and also the requirements for this course.

Title IX and Pregnancy Related Conditions

Grinnell College is committed to compliance with Title IX and to supporting the academic success of pregnant and parenting students and students with pregnancy related conditions. If you are a pregnant student, have pregnancy related conditions, or are a parenting student (child under one-year needs documented medical care) who wishes to request reasonable related supportive measures from the College under Title IX, please email the Title IX Coordinator at titleix@grinnell.edu. The Title IX Coordinator will work with Disability Resources and your professors to provide reasonable supportive measures in support of your education while pregnant or as a parent under Title IX.

Academic Advising Support

If you have other needs that aren't addressed through the above resources, please let me know soon so that we can work together for the best possible learning environment. In some cases, I will recommend consulting with the Academic Advising staff. They are an excellent resource for developing strategies for academic success and can connect you with other campus resources as well:

http://www.grinnell.edu/about/offices-services/academic-advising. If I notice that you are encountering difficulty, in addition to communicating with you directly about it, I will also likely submit an academic alert via Academic Advising's SAL portal. This reminds you of my concern, and it notifies the Academic Advising team and your adviser(s) so that they can reach out to you with additional offers of support.

Academic Honesty

Please familiarize yourself with Grinnell's policies on academic honesty. Below are some additional policies for this course.

You are allowed (and strongly encouraged) to ask me for clarification and hints, to attend mentor sessions to ask Ian about homework, and to brainstorm homework ideas with your classmates. But you must write your final solution on your own. You will be required to list anyone you worked with on your homework submissions.

If you find an alternative textbook, article, video, or any other resource that helps you with the material on a homework assignments, then you must mention that on your homework. I recommend checking with me before using any external resources to make sure they fit the level of our course. Note you must cite any results that you use from our textbook(s) as well.

You are not allowed to consult external resources that provide proofs or partial proofs of your homework statements. This includes other textbooks that provide solutions/proofs of the homework statements, online forums, any AI tools or large language models (LLMs) such as Chat GPT, or any other external resource that gives you a partial or complete solution to one of your assigned problems.

You should not be using AI tools or LLMs for any reason while coming up with the mathematical ideas or writing style in your assignments. You are not allowed to use these tools to get solutions (partial or complete), hints, help you brainstorm, or aid with writing. The only exception is that you can consult these tools or online forums for help with LaTeX commands and formatting (e.g. "how do I make matrix in latex").

Consulting any completed or partial solution is academically dishonest. A key goal in any math course is to learn how to navigate through being stuck. Reading and understanding a math solution is much easier than figuring it out yourself. It hurts your learning to find solutions instead of coming up with them. **I will intentionally assign some more difficult problems so that you can experience working through being stuck, and I do not assume students will ace every homework assignment.** If you're feeling stuck and really don't know how to proceed, then you should reach out to me and we can come up with a plan together.

Some clarifications on collaborating with other students

You are encouraged to work with your classmates on problems, but you should not share solutions or hints to problems you've already solved. To clarify, if you are brainstorming how to solve a problem with a group of students, and in the course of your conversation someone suggests a key

piece of the solution, then everyone in your study group is welcome to use that key piece when you write up your own proofs later on. Additionally, you are allowed to discuss your written solutions for this problem with that group (since you already have an agreed-upon path through the proof that you discovered together).

On the other hand, if someone has already solved a problem (either on their own or with a different group), then it is not okay to share key pieces with other students in the class. It's very difficult to differentiate between a "key piece" and "just a hint". Thus if someone asks for help or hints on a homework problem that you've already solved, then you should tell them that you already solved the problem and instead recommend they attend office hours or a mentor session if they are stuck.

If you do decide to use LaTeX then note **you should never share your LaTeX code for a solution with another student.** Even if you worked together, you should not be sharing full or partial solutions in this way. If you or a classmate is struggling to learn LaTeX, then they should come chat with me to come up with a plan. It is okay to share how to do things in LaTeX (e.g. "you can make a 2×2 matrix by typing `\begin{pmatrix} a&b\\ c&d \end{pmatrix}`" as opposed to "here's my solution to Problem 3 as an example for how to type this").

Class Schedule

See PWeb for a daily course schedule with readings and topics covered in class.

Important Class Days and Dates

- Every Wednesday at 11:59PM (except exam weeks)... Part 1 of Homework due on Gradescope
- Every Tuesday at 8:30AM... Part 2 of Homework due in class (unless you use LaTeX, then you can email a pdf of the assignment by 4PM)
- Every Thursday at 11:59PM (except exam weeks)... reading reflection due on PWeb
- Thursday, February 20th in class... Exam 1
- Thursday, March 27th in class... Exam 2
- Thursday, April 24th in class... Exam 3
- Wednesday, May 14th 9AM-12PM... Final Exam